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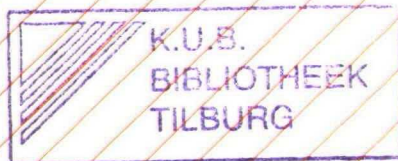
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DEPARTMENT OF ECONOMICS
RESEARCH MEMORANDUM

**FREE END-POINT LINEAR-QUADRATIC
CONTROL SUBJECT TO IMPLICIT
CONTINUOUS-TIME SYSTEMS: NECESSARY AND
SUFFICIENT CONDITIONS FOR SOLVABILITY**
Ton Geerts

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Optimal Control

Communicated by Prof.dr. J.M. Schumacher

FREE END-POINT LINEAR-QUADRATIC CONTROL
SUBJECT TO IMPLICIT CONTINUOUS-TIME SYSTEMS:
NECESSARY AND SUFFICIENT CONDITIONS FOR SOLVABILITY

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Abstract.

For an *implicit* continuous-time system with arbitrary constant coefficients we derive necessary and sufficient conditions for solvability of the associated free end-point linear-quadratic optimal control problem. In particular, this problem turns out to be solvable if and only if the underlying system is output stabilizable, as is the case for a *standard* system.

Keywords.

Implicit systems, linear-quadratic problems, regularity, impulsive-smooth distributions, output stabilizability.

1. Introduction and preliminaries.

Given the implicit continuous-time system Σ :

$$E\dot{x}(t) = Ax(t) + Bu(t), \quad (1.1a)$$

$$y(t) = Cx(t) + Du(t), \quad (1.1b)$$

with $u(t) \in \mathbb{R}^m$, $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^r$ for all $t \in \mathbb{R}^+ := [0, \infty)$. Let k denote the number of equations in (1.1a) and let $e = \text{rank}(E)$. All matrices involved are real-valued and constant. We may, and hence will, assume that $[E \ A \ B]$ is of full row rank. If E is invertible, then the solutions of (1.1a) are

$$x(t) = \exp(E^{-1}At)x_0 + \int_0^t \exp(E^{-1}A(t-\tau))E^{-1}Bu(\tau)d\tau \quad (1.2)$$

($x_0 \in \mathbb{R}^n$ arbitrary) and hence every x_0 is consistent, i.e., for

every x_0 , (1.1a) has a solution x with $x(0^+) = x_0$. If E is not invertible, however, this need not be the case and inconsistent initial conditions may give rise to impulsive solutions of (1.1a), see e.g. [12], [2]. The most natural way to deal with such phenomena is the use of distributions [11], as was done earlier in e.g. [2]. Instead of (1.1), we will consider its distributional interpretation:

$$E\delta^{(1)} * x = Ax + Bu + Ex_0\delta, \quad (1.3a)$$

$$y = Cx + Du, \quad (1.3b)$$

where δ , $\delta^{(1)}$ denote the Dirac distribution and its distributional derivative, respectively, $*$ stands for convolution of distributions, $x_0 \in \mathbb{R}^n$, arbitrary. Moreover, $u \in \mathcal{C}_{\text{imp}}^m$, the m -vector version of \mathcal{C}_{imp} , the commutative algebra (over \mathbb{R}) of *impulsive-smooth* distributions [10, Def. 3.1], [9]. A distribution is *impulsive-smooth* if it can be decomposed (uniquely) in an *impulse* (any linear combination of δ and its derivatives $\delta^{(i)}$, $i \geq 1$) and a *smooth* distribution. A distribution is called *smooth* if it corresponds to a function that is smooth on \mathbb{R}^+ and zero elsewhere. Let \mathcal{C}_{sm} denote the subalgebra of smooth distributions. The distributional derivative of $u \in \mathcal{C}_{\text{sm}}$, $u^{(1)} = \delta^{(1)} * u$, equals $\dot{u} + u(0^+)\delta$, where $\dot{u} \in \mathcal{C}_{\text{sm}}$ denotes the ordinary derivative of u on \mathbb{R}^+ . Example: Let $u \in \mathcal{C}_{\text{sm}}$ correspond to $2\exp(t)$ on \mathbb{R}^+ . Then $u^{(1)} = \dot{u} + 2\delta$. For more details on \mathcal{C}_{imp} , see [9] - [10], also [6] - [8]; because of its nice properties we can keep our treatment fully *algebraic*. It can be readily shown that, for every real-valued square matrix H , $(I\delta^{(1)} - H\delta)$ is invertible (w.r.t. convolution); its inverse corresponds to $\exp(Ht)$ on \mathbb{R}^+ . Hence the solutions of (1.3a) reduce to the ordinary ones ((1.2)) if E is invertible and $u \in \mathcal{C}_{\text{sm}}^m$; for every pair (x_0, u) , (1.3a) has exactly one solution. Also, note that (1.3a) reduces to (1.1a) if u and x are smooth. In general, however, the *solution set*

$$S(x_0, u) = \{x \in \mathcal{C}_{\text{imp}}^n \mid [E\delta^{(1)} - A\delta] * x = Bu + Ex_0\delta\}, \quad (1.4)$$

may be empty or contain infinitely many elements, see [6]. We are ready for the definition of the free end-point linear-quadratic control problem subject to (1.3).

(LQCP)⁻: For all x_0 , determine

$$J^-(x_0) := \inf \left\{ \int_0^\infty Y' y dt \mid u \in C_{sm}^m, x \in S(x_0, u) \cap C_{sm}^n \right\}, \quad (1.5)$$

and if, for every x_0 , $J^-(x_0) < \infty$, then compute (if possible) optimal controls $\bar{u} \in C_{sm}^m$ and associated optimal state trajectories $\bar{x} \in S(x_0, \bar{u})$. The problem (LQCP)⁻ is *solvable* if both requirements are met.

In the sequel we will need several subspaces of interest. Let

$$\begin{aligned} \mathcal{F}(\Sigma) &:= \{x_0 \in \mathbb{R}^n \mid \exists u \in C_{sm}^m \exists x \in S(x_0, u) \cap C_{sm}^n : \lim_{t \rightarrow \infty} \begin{bmatrix} u(t) \\ x(t) \end{bmatrix} = 0\}, \\ \mathcal{F}_C(\Sigma) &:= \{x_0 \in \mathbb{R}^n \mid \exists u \in C_{sm}^m \exists x \in S(x_0, u) \cap C_{sm}^n : y = 0, \\ &\quad x(0^+) = x_0\}, \end{aligned}$$

$$\mathcal{O}(\Sigma) := \{x_0 \in \mathbb{R}^n \mid \exists u \in C_{sm}^m \exists x \in S(x_0, u) \cap C_{sm}^n : \lim_{t \rightarrow \infty} y(t) = 0\} \quad (1.6)$$

and let $\mathcal{F}_B(\Sigma)$, $\mathcal{O}_B(\Sigma)$ denote those subspaces of $\mathcal{F}(\Sigma)$ and $\mathcal{O}(\Sigma)$, for which u and x in the respective definitions are of the Bohl type (a Bohl function is any linear combination of functions $t^k \exp(\lambda t)$, $k \geq 0$). For $\mathcal{F}_C(\Sigma)$ we have the following result.

Proposition 1.1 [7, Prop. 3.5, Theorem 3.6].

$\mathcal{F}_C(\Sigma)$ is the largest subspace $\mathcal{L} \subset \mathbb{R}^n$ for which there exists a matrix $F \in \mathbb{R}^{m \times n}$ such that $(A + BF)\mathcal{L} \subset E\mathcal{L}$, $(C + DF)\mathcal{L} = 0$.

If, moreover,

$$\mathcal{V}(\Sigma) := \{x_0 \in \mathbb{R}^n \mid \exists u \in C_{sm}^m \exists x \in S(x_0, u) \cap C_{sm}^n : y = 0\}, \quad (1.7)$$

then [7, Prop. 3.4] tells us that

$$\mathcal{V}(\Sigma) = \mathcal{F}_C(\Sigma) + \ker(E). \quad (1.8)$$

In [10], [7] a point $x_0 \in \mathcal{V}(\Sigma)$ is called *weakly unobservable*; we establish that all points in $\mathcal{V}_C(\Sigma)$ are also *consistent*. Let, for any subspace T and η any complex row vector of compatible size, ηT stand for $\{\eta t \mid t \in T\}$. The next result is stated in [3].

Proposition 1.2.

Let E be invertible. Then $\mathcal{V}(\Sigma) + \mathcal{V}(\Sigma) = \mathcal{O}(\Sigma) = \{x_0 \in \mathbb{R}^n \mid J^-(x_0) < \infty\}$, $\mathcal{O}_B(\Sigma) = \mathcal{O}(\Sigma)$, $\mathcal{V}_B(\Sigma) = \mathcal{V}(\Sigma)$ and $\mathcal{O}(\Sigma) = \mathbb{R}^n$ if and only if, for all $\lambda \in \mathbb{C}$ with $\operatorname{Re}(\lambda) \geq 0$,

$$\eta[\lambda E - A, -B] = 0 \text{ and } \eta E \mathcal{V}(\Sigma) = 0 \text{ only if } \eta = 0. \quad (1.9)$$

If in Proposition 1.2, $C = I$ and $D = 0$, then $\mathcal{V}(\Sigma) = 0$ and we reobtain the well-known statement that $\mathcal{V}(\Sigma) = \mathbb{R}^n$ if and only if Σ is (state) stabilizable. We will say that Σ is *output stabilizable* if $\mathcal{O}(\Sigma) = \mathbb{R}^n$.

Now, we consider Σ with arbitrary E . From [6, Theorem 4.5] we borrow

Proposition 1.3.

$$\begin{aligned} \forall x_0 \in \mathbb{R}^n \exists u \in \mathbb{C}_{sm}^m \exists x \in S(x_0, u) \cap \mathbb{C}_{sm}^n &\Leftrightarrow \\ \operatorname{im}(E) + \operatorname{im}(B) + A(\ker(E)) &= \mathbb{R}^1. \end{aligned} \quad (1.10)$$

2. Main results. Without loss of generality, we may rewrite Σ in the form

$$\begin{aligned} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \delta^{(1)} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u + \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{01} \\ x_{02} \end{bmatrix} \delta, \\ y &= [C_1 \ C_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + Du. \end{aligned} \quad (2.1)$$

Assume that (1.10) is satisfied, i.e., that $[A_{22} \ B_2]$ is of full row rank. Let $T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \in \mathbb{R}^{(n+m-e) \times (n+m-k)}$, of full column rank, be such that $[A_{22} \ B_2]T = 0$. Set $N := A_{22}A_{22}' + B_2B_2' > 0$, $L := T'T > 0$. Then

$$Q := \begin{bmatrix} A_{22}' & T_1 \\ B_2' & T_2 \end{bmatrix} \text{ is invertible, } Q^{-1} = \begin{bmatrix} N^{-1} & 0 \\ 0 & L^{-1} \end{bmatrix} Q'. \quad (2.2)$$

If \bar{z} denotes the standard system

$$\delta^{(1)} * z = \bar{A}z + \bar{B}v + z_0\delta, \quad (2.3a)$$

$$w = \bar{C}z + \bar{D}v, \quad (2.3b)$$

with

$$\begin{aligned} \bar{A} &:= A_{11} - [A_{12} \ B_1] \begin{bmatrix} A_{22}' \\ B_2' \end{bmatrix} N^{-1} A_{21}, \quad \bar{B} := [A_{12} \ B_1] T, \\ \bar{C} &:= C_1 - [C_2 \ D] \begin{bmatrix} A_{22}' \\ B_2' \end{bmatrix} N^{-1} A_{21}, \quad \bar{D} := [C_2 \ D] T, \end{aligned} \quad (2.3c)$$

then it turns out that all solutions for (1.3) can be expressed in solutions for (2.3) and vice versa.

Theorem 2.1.

Let $\begin{bmatrix} x_{01} \\ x_{02} \end{bmatrix} \in \mathbb{R}^n$, $u \in C_{\text{imp}}^m$ and $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in S(\begin{bmatrix} x_{01} \\ x_{02} \end{bmatrix}, u)$. Then

$$x_1 = z(x_{01}, v), \quad \begin{bmatrix} x_2 \\ u \end{bmatrix} = \begin{bmatrix} A_{22}' \\ B_2' \end{bmatrix} N^{-1} (-A_{21}) (z(x_{01}, v)) + Tv$$

with $v = L^{-1}[T_1'x_2 + T_2'u] \in C_{\text{imp}}^{n+m-k}$. Moreover, $y = w(x_{01}, v)$.

Conversely, let $z_0 \in \mathbb{R}^e$, $v \in C_{\text{imp}}^{n+m-k}$, and $z = z(z_0, v)$. Then $u = -B_2'N^{-1}A_{21}z + T_2v \in C_{\text{imp}}^m$ and, for all x_{02} , $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in S(\begin{bmatrix} z_0 \\ x_{02} \end{bmatrix}, u)$ with $x_1 = z$ and $x_2 = -A_{22}'N^{-1}A_{21}z + T_1v$. In addition, $y = w(z_0, v)$.

Proof. First half. If in (2.3a) with $z_0 = x_{01}$ we insert v as

prescribed, then $\delta^{(1)} * z = \bar{A}z + [A_{12} \ B_1]Q \begin{bmatrix} N^{-1} & 0 \\ 0 & L^{-1} \end{bmatrix} \{Q' \begin{bmatrix} x_2 \\ u \end{bmatrix} + \begin{bmatrix} A_{21}'x_1 \\ 0 \end{bmatrix}\} + x_{01}\delta = \bar{A}z + [A_{12} \ B_1] \begin{bmatrix} x_2 \\ u \end{bmatrix} + (A_{11} - \bar{A})x_1 + x_{01}\delta = \bar{A}z + (\delta^{(1)} * x_1 - A_{11}x_1 - x_{01}\delta) + (A_{11} - \bar{A})x_1 + x_{01}\delta = \delta^{(1)} * x_1 + \bar{A}(z - x_1)$, by (2.1) - (2.2). Hence $[I_e \delta^{(1)} - \bar{A}\delta] * (z - x_1) = 0$

and $z - x_1 = 0$. Since $\begin{bmatrix} x_2 \\ u \end{bmatrix} = QQ^{-1} \begin{bmatrix} x_2 \\ u \end{bmatrix} = \begin{bmatrix} A_{22}' \\ B_2' \end{bmatrix} N^{-1} (-A_{21}x_1) + Tv$, the rest is clear. The second half is now trivial.

Observe that if in (2.1), $e = k$ (i.e., E is of full row rank),

then T is invertible and $\bar{A} = A_{11}$, $\bar{C} = C_1$ in (2.3). Here is our first main result.

Theorem 2.2

If the system (1.3) satisfies (1.10), then $\varphi(\Sigma) + \psi(\Sigma) = \sigma(\Sigma) = \{x_0 \in \mathbb{R}^n \mid J^-(x_0) < \infty\}$, $\varphi_B(\Sigma) = \varphi(\Sigma)$ and $\sigma_B(\Sigma) = \sigma(\Sigma)$. Moreover, (1.3) is output stabilizable if and only if (1.9) - (1.10) are satisfied.

Proof. Consider (2.1) - (2.3). Then $[\eta_1 \ \eta_2] \begin{bmatrix} \lambda I - A_{11} & -A_{12} & -B_1 \\ & -A_{21} & -A_{22} & -B_2 \end{bmatrix} = 0$ if and only if $\eta_1[\lambda I_e - \bar{A}, -\bar{B}] = 0$ and η_2 equals $-\eta_1[A_{12} \ B_1] \begin{bmatrix} A_{22} & B_2 \\ B_2^* & D \end{bmatrix}^{-1}$, for every $\lambda \in \mathbb{C}$. Since $\ker(E)$ is contained in all subspaces involved, both claims follow immediately from Props. 1.2, 1.3 and Theorem 2.1.

Now, let us consider $(LQCP)^-$. By Theorem 2.2, it is obvious that output stabilizability is *necessary* for solvability. Thus, assume that Σ is output stabilizable. It follows from Theorem 2.1 that $(LQCP)^-$ is solvable if and only if the corresponding problem subject to (2.3) is solvable. Since $\bar{\Sigma}$ is output stabilizable, it is known that the optimal cost for the latter *standard* problem can be represented as a quadratic form [3] - [4]. Moreover, for every initial condition z_0 there exist a unique optimal control v and a unique optimal state trajectory z , both of the Bohl type, if $\ker(\bar{D}) = 0$, i.e., if the problem is *regular* [4]. In other words, output stabilizability of $\bar{\Sigma}$ is also *sufficient* for solvability of the free end-point LQCP subject to (2.3) if the problem is regular. If the problem is *singular*, then for every z_0 optimal controls and state trajectories still exist in the sense that the infimum of the cost criterion (1.5) is actually attained - yet, in general they are distributions rather than functions [13], [5]. Note that, in terms of (2.1) - (2.3), $\ker(\bar{D}) = 0 \Leftrightarrow \ker \begin{pmatrix} A_{22} & B_2 \\ C_2 & D \end{pmatrix} = 0$. Hence, by Theorem 2.1, we arrive at

Theorem 2.3.

For every $x_0 \in \mathbb{R}^n$, $J^-(x_0) < \infty$ if and only if (1.9) - (1.10) are satisfied. Assume this to be the case. Then there exists a unique real symmetric matrix $P^- \geq 0$ with $\ker(E) \subset \ker(P^-)$, such that, for all x_0 , $J^-(x_0) = x_0' P^- x_0$. If

$$\ker \begin{pmatrix} E & 0 \\ C & D \end{pmatrix} \cap [A \ B]^{-1} \text{im}(E) = 0, \quad (2.4)$$

then for every x_0 there exists a unique optimal control \bar{u} and a unique optimal state trajectory $\bar{x} \in S(x_0, \bar{u})$, both of the Bohl type. If (2.4) is not satisfied, then for every x_0 there exist $u \in C_{\text{imp}}^m$ and $x \in S(x_0, u)$ such that $y \in C_{\text{sm}}^r$ and $J^-(x_0) = \int_0^\infty y' y dt$.

The condition (2.4) can be interpreted as a system property for Σ . In [8, Theorem 3.2] it is proven that (2.4) holds if and only if

$$y \in C_{\text{sm}}^r \Leftrightarrow u \in C_{\text{sm}}^m, \ x \in S(x_0, u) \cap C_{\text{sm}}^n. \quad (2.5)$$

In other words, (2.4) stands for the property that outputs for Σ are *functions* only if associated controls and state trajectories are *functions* as well. Therefore $(\text{LQCP})^-$ is called *regular* in [8] if (2.5) is satisfied. Observe that (2.4) reduces to the classical $\ker(D) = 0$ if E is invertible. The linear-quadratic problems considered in [1] - [2] are regular in the sense of (2.4), since it is assumed there that $\ker \begin{pmatrix} E & 0 \\ C & D \end{pmatrix} = 0$. An example of a regular linear-quadratic problem for which $\ker \begin{pmatrix} E & 0 \\ C & D \end{pmatrix} \neq 0$ is given in [8]. Also, note that, unlike in [1] - [2], we allow the state trajectories to diverge.

If Σ is output stabilizable and (2.4) is not satisfied, then we may still assume $\begin{bmatrix} E & 0 \\ A & B \\ C & D \end{bmatrix}$ to be of full column rank. Let this be the case. Now the distributional optimal controls and state trajectories for (LQCP)⁻ (see Theorem 2.3) are in general not unique. This follows from Theorem 2.1, since it is proven in [5] that optimal controls and state trajectories for (LQCP)⁻ subject to a standard system Σ are unique if and only if Σ is left invertible [10, Theorem 3.26], i.e., if in (1.3) with E invertible, $y = 0$ and $x_0 = 0$ imply that $u = 0$ (and hence also $x = 0$). Moreover, the smooth parts of these unique optimal controls and state trajectories are of the Bohl type.

Two different concepts for left-invertibility for *implicit* systems are given in [7]. There, a system (1.3) is defined left invertible in the strong sense if $x_0 = 0$ and $y = 0$ imply that $u = 0$ and $Ex = 0$ (and left invertible in the weak sense if merely $u = 0$), see [7, Defs. 4.1, 4.10]. Under the above-mentioned rank condition, it is proven in [7, Corollary 4.15] that Σ is left invertible in the strong sense if and only if $x_0 = 0$, $y = 0$ imply that $u = 0$, $x = 0$. Hence, again by Theorem 2.1, Σ is left invertible in the strong sense if and only if (2.3) is left invertible in the sense of [10] and thus

Corollary 2.4.

Let Σ be output stabilizable and $\ker \begin{pmatrix} E & 0 \\ A & B \\ C & D \end{pmatrix} = 0$. Then for every x_0 there exists exactly one (possibly distributional) \bar{u} and exactly one (possibly distributional) \bar{x} such that $\bar{y} \in c_{sm}^r$ and $\int_0^\infty \bar{y}' \bar{y} dt = J^-(x_0)$ if and only if Σ is left invertible in the strong sense. Moreover, if \bar{u}_2 , \bar{x}_2 denote the smooth parts of \bar{u} and \bar{x} , then \bar{u}_2 and \bar{x}_2 are of the Bohl type.

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- 466 Prof.Dr. Th.C.M.J. van de Klundert - Prof.Dr. A.B.T.M. van Schaik
Economische groei in Nederland in een internationaal perspectief
- 467 Dr. Sylvester C.W. Eijffinger
The convergence of monetary policy - Germany and France as an example
- 468 E. Nijssen
Strategisch gedrag, planning en prestatie. Een inductieve studie binnen de computerbranche
- 469 Anne van den Nouweland, Peter Borm, Guillermo Owen and Stef Tijs
Cost allocation and communication
- 470 Drs. J. Grazell en Drs. C.H. Veld
Motieven voor de uitgifte van converteerbare obligatieleningen en warrant-obligatieleningen: een agency-theoretische benadering
- 471 P.C. van Batenburg, J. Kriens, W.M. Lammerts van Bueren and R.H. Veenstra
Audit Assurance Model and Bayesian Discovery Sampling
- 472 Marcel Kerkhofs
Identification and Estimation of Household Production Models
- 473 Robert P. Gilles, Guillermo Owen, René van den Brink
Games with Permission Structures: The Conjunctive Approach
- 474 Jack P.C. Kleijnen
Sensitivity Analysis of Simulation Experiments: Tutorial on Regression Analysis and Statistical Design
- 475 C.P.M. van Hoesel
An $O(n \log n)$ algorithm for the two-machine flow shop problem with controllable machine speeds
- 476 Stephan G. Vanneste
A Markov Model for Opportunity Maintenance
- 477 F.A. van der Duyn Schouten, M.J.G. van Eijs, R.M.J. Heuts
Coordinated replenishment systems with discount opportunities
- 478 A. van den Nouweland, J. Potters, S. Tijs and J. Zarzuelo
Cores and related solution concepts for multi-choice games
- 479 Drs. C.H. Veld
Warrant pricing: a review of theoretical and empirical research
- 480 E. Nijssen
De Miles and Snow-typologie: Een exploratieve studie in de meubelbranche
- 481 Harry G. Barkema
Are managers indeed motivated by their bonuses?

- 482 Jacob C. Engwerda, André C.M. Ran, Arie L. Rijkeboer
Necessary and sufficient conditions for the existence of a positive definite solution of the matrix equation $X + A^T X^{-1} A = I$
- 483 Peter M. Kort
A dynamic model of the firm with uncertain earnings and adjustment costs
- 484 Raymond H.J.M. Gradus, Peter M. Kort
Optimal taxation on profit and pollution within a macroeconomic framework
- 485 René van den Brink, Robert P. Gilles
Axiomatizations of the Conjunctive Permission Value for Games with Permission Structures
- 486 A.E. Brouwer & W.H. Haemers
The Gewirtz graph - an exercise in the theory of graph spectra
- 487 Pim Adang, Bertrand Melenberg
Intratemporal uncertainty in the multi-good life cycle consumption model: motivation and application
- 488 J.H.J. Roemen
The long term elasticity of the milk supply with respect to the milk price in the Netherlands in the period 1969-1984
- 489 Herbert Hamers
The Shapley-Entrance Game
- 490 Rezaul Kabir and Theo Vermaelen
Insider trading restrictions and the stock market
- 491 Piet A. Verheyen
The economic explanation of the jump of the co-state variable
- 492 Drs. F.L.J.W. Manders en Dr. J.A.C. de Haan
De organisatorische aspecten bij systeemontwikkeling een beschouwing op besturing en verandering
- 493 Paul C. van Batenburg and J. Kriens
Applications of statistical methods and techniques to auditing and accounting
- 494 Ruud T. Frambach
The diffusion of innovations: the influence of supply-side factors
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factors
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van keurmerken
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normal distributed errors
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distribution
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Renewal theoretic background
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On Coloring j-Unit Sphere Graphs

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Basics of inventory management: Part 2
The (R,S)-model
- 522 A.G. de Kok
Basics of inventory management: Part 3
The (b,Q)-model
- 523 A.G. de Kok
Basics of inventory management: Part 4
The (s,S)-model
- 524 A.G. de Kok
Basics of inventory management: Part 5
The (R,b,Q)-model
- 525 A.G. de Kok
Basics of inventory management: Part 6
The (R,s,S)-model
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een management-georiënteerde benadering
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decisions
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